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A SIMPLE MODEL OF THE INTERPLANETARY MAGNETIC FIELD

PART II: THE COSMIC RAY ANISOTROPY

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SUMMARY

Three explanations for the observed cosmic ray anisotropy are investigated. The possibility that the anisotropy is due to trapped orbits in the interplanetary magnetic field is explored by analyzing the motion of charged particles in the stretched dipole field developed in Part I of this study, published separately. It is found that an anisotropy is possible, but only when several unlikely conditions are met. Two other theories of the anisotropy, ascribing it to a sunward flux density gradient or to the Compton-Getting effect, are then discussed. It is shown that in general both effects occur together; for conservative fields they cancel each other and no anisotropy occurs, as indeed might be expected from Liouville's theorem. Consequently, any gradient of cosmic ray flux density which might be measured in interplanetary space is not necessarily connected with the observed anisotropy.

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INTRODUCTION

Because of their large gyration radii in the interplanetary field, cosmic ray particles are likely to reflect the gross structure of this field rather than local irregularities. One of the important properties of these particles is their anisotropy, which manifests itself in a solar daily variation of about 0.3 percent, observed from the surface of the earth. The direction of the maximum flux is approximately tangential to the earth's orbit on the afternoon side, and the range of energies at which the anisotropy has been observed is roughly 7-20 Bev. In this range the relative modulation is nearly independent of energy (Reference 1), in sharp contrast to other types of cosmic ray intensity variations, which generally decrease rapidly with increasing energy.

Various theories have been advanced to explain this anisotropy, most of which fall into one of the following classes:

1. Theories based on the Störmer effect. According to these the anisotropy is caused by a solar-centered magnetic field, in a manner similar to that by which the terrestial dipole field creates an east-west anisotropy in cosmic radiation observed near the earth's surface.

2. Theories based on a gradient of cosmic ray flux density in the direction of the sun.

3. Theories based on the Compton-Getting effect. These assume that the radiation is isotropic in some frame of reference moving relative to the earth.

In what follows, these three approaches (in the above order) will be investigated in more detail.

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†The major part of this work was performed while the author held a National Academy of Sciences - National Research Council Post - Doctoral Resident Research Associateship.

STÖRMER EFFECT THEORIES

It often has been suggested that the cosmic ray anisotropy is caused by an interplanetary dipole field (References 2-6). Essentially, the idea is that a weak scattering mechanism operates to fill the trapped orbits which, however, are less densely populated than free ones, because of some additional loss mechanism, for example, scattering into orbits hitting the sun (Reference 5). In the energy range where some of the radiation received on the earth is trapped and the rest arrives directly, an anisotropy will be observed, with the maximum effect in the direction normal to the planes of the lines of force.

In view of the stretching and twisting expected of the (average) interplanetary field, this model has to be modified considerably. Accordingly, the motion of charged particles in the stretched dipole field, developed as a worked example in Part I of this study (Reference 7), will now be investigated. As in Part I of this work (Reference 7) space is assumed to be divided into three regions by two concentric spheres of radii R_0 and R_1 (Figure 1). Region I is assumed to rotate rigidly with angular velocity ω. It contains the source of the magnetic field, assumed to be concentrated at the origin. Region II contains a compressible conducting fluid flowing radially outward. In region III, which extends to infinity, no motion takes place. In actuality region I represents the sun and region II the space swept by the solar wind. The magnetic field resulting (at high conductivity) from an axial dipole source is given by Figure 2 (and by Equations 3 and 4 below).

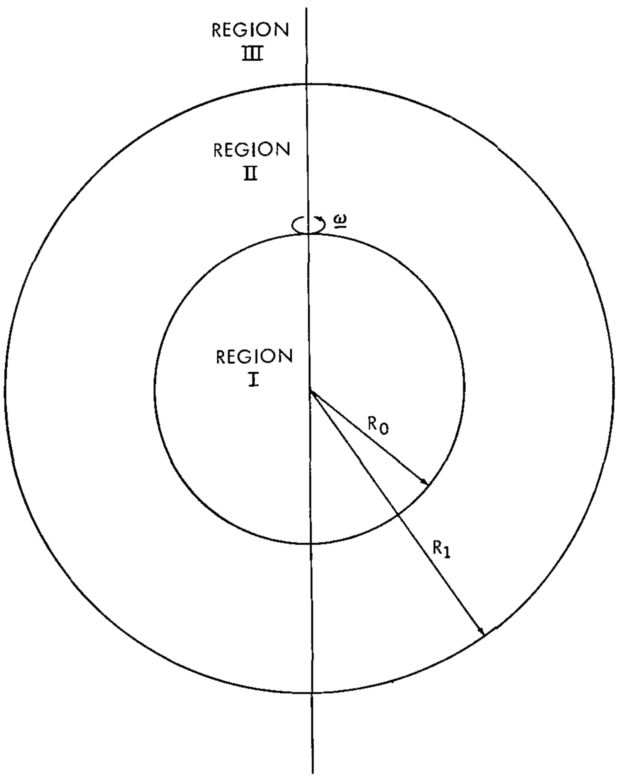

Figure 1—The division of space into three regions.

The Lagrangian for the motion of a particle with rest mass m, velocity v, and charge q, in an electromagnetic field with magnetic and electric potentials **A** and Ψ_0, is

$$L = -mc^2\left(1-\frac{v^2}{c^2}\right)^{1/2} + q(\mathbf{A}\cdot\mathbf{v}) - q\Psi_0 \quad , \qquad (1)$$

If neither **A** nor Ψ_0 depends on the azimuth angle ϕ, then $\partial L/\partial\dot{\phi} = \chi_0$ is a constant of motion ("Stormer's first integral"). If p denotes the particle's momentum and ω the angle between its velocity and the direction of ϕ, the following expression is obtained:

$$\chi_0 = r\sin\theta\left(p\cos\omega + qA_\phi\right) \qquad (2)$$

Because of the electric potential Ψ_0, p is not conserved. However, the energies of interest here are considerably larger than the

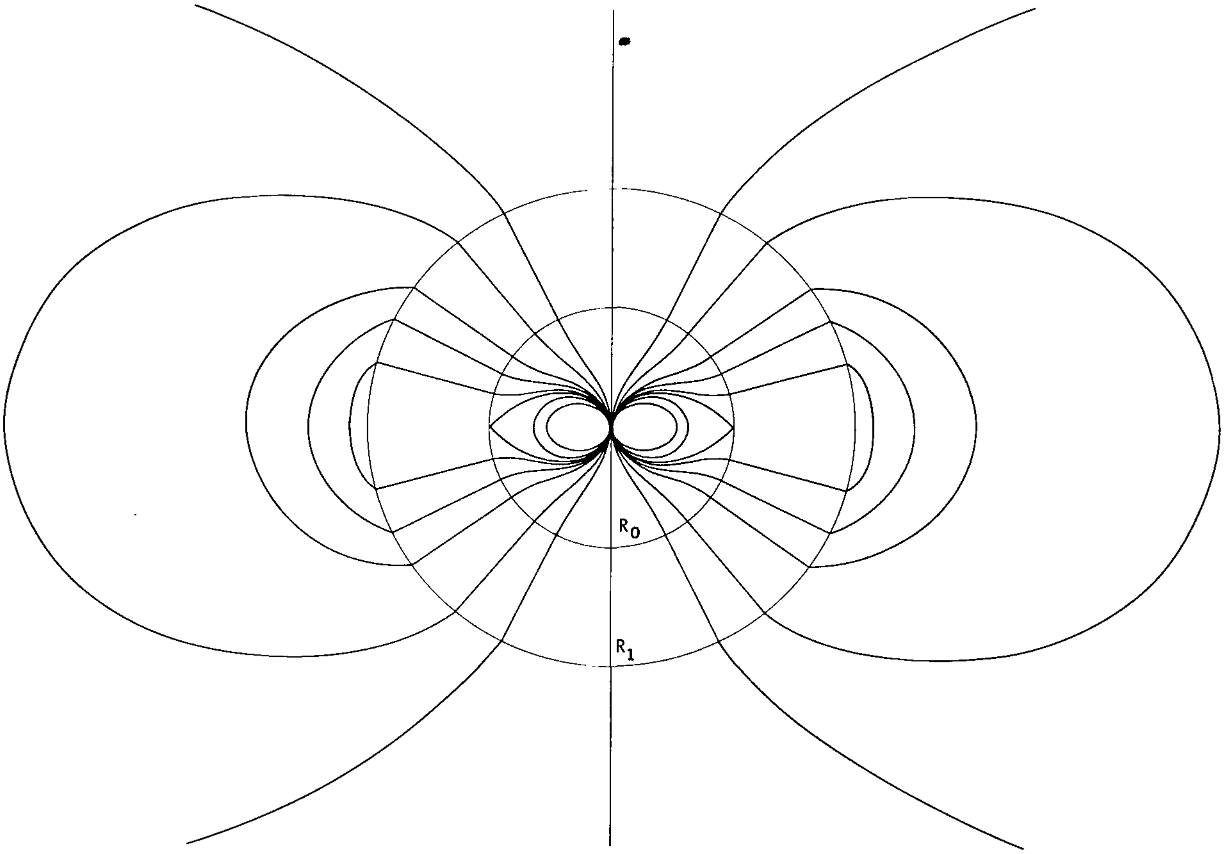

Figure 2—Lines of force of the magnetic field, for the limit of very high conductivity.

changes they undergo in the electric field, so that, in order to simplify the calculation, p will be considered constant. From the example of Part I (Reference 7) it follows that, in the limit of high conductivity with a source of dipole moment M,

$$A_\phi = \frac{3\mu_0 M}{8\pi R_0} \frac{\sin\theta}{r} \tag{3}$$

for region II, and

$$A_\phi = \frac{3\mu_0 M R_1}{8\pi R_0} \frac{\sin\theta}{r^2} \tag{4}$$

for region III (Figure 2).

Only the poloidal component of B contributes to Equation 2. As in the treatment of motion in an ordinary dipole field (References 8 and 9), all lengths will be measured in momentum-dependent

"Störmer units,"

$$1_{st} = \left(\frac{3qM\mu_0 R_1}{8\pi R_0 p}\right)^{1/2}$$

$$= (R_0 R_1)^{1/2} \left(\frac{qB_s}{p} R_0\right)^{1/2} \tag{5}$$

The first factor on the bottom right is a mean value of r in region II. The second is the square root of the ratio between R_0 and the radius of gyration obtained, without the toroidal component, on the inner boundary of region II at its intersection with the equatorial plane. By choosing the field intensity at this point as $B_s = 1$ gauss, $q = 1.6 \times 10^{-19}$ coulomb, and $R_0 = 7 \times 10^{10}$ cm (the solar radius), and by measuring p in Bev/c,

$$1_{st} = 145\left(\frac{R_0 R_1}{p}\right)^{1/2} . \tag{6}$$

By measuring all lengths in Störmer units and defining

$$\chi_1 = \chi_0 \left(\frac{8\pi R_0}{3\mu_0 MR_1 qp}\right)^{1/2}$$

Equation 2 becomes

$$\left.\begin{aligned} \cos\omega &= \frac{1}{r}\left(\frac{\chi_1}{\sin\theta} - \frac{\sin\theta}{R_1}\right) , \\ \cos\omega &= \frac{\chi_1}{r\sin\theta} - \frac{\sin\theta}{r^2} , \end{aligned}\right\} \tag{7}$$

for regions II and III, respectively.

We shall consider those particles with the given momentum p and invariant χ_1. In general in certain parts of the (r, θ) plane Equations 7 yield $|\cos\omega| > |$. Therefore these parts of the plane are not accessible to these particles. The rest of the plane forms the "allowed region," and if this region is multiply connected trapping may occur.

The allowed region in region II is simply connected. To see this it is best to consider a single radial direction with fixed θ. For all points having this θ, the term $(\chi_1/\sin\theta)-(\sin\theta/R_1)$ in Equations 7 is constant and $|\cos\omega|$ is a monotonic single-valued function of r. As the origin is approached, $|\cos\omega|$ increases steadily until forbidden region is reached $\left[\text{for any } \theta \text{ except } \theta = \arcsin(\chi_1 R_1)^{1/2} \text{ where the allowed region extends to the origin}\right]$. Thus the forbidden regions cluster around the origin and the allowed region is simply connected.

The allowed region of the dipole field in region III, on the other hand, is multiply connected if $\chi_1 > 2$ (References 8 and 9). This region — with which the allowed part of region II merges smoothly — consists then of an inner "trapped" region in which $r < 1$ everywhere and a "free" region where $r > 1$. Region III will include part of the trapped region only if $R_1 < 1$ or, by Equation 6,

$$\frac{R_1}{R_0} < \frac{21,000}{p} \quad . \tag{8}$$

We shall assume that Equation 8 holds and consider orbits in the equatorial plane of region II. As a simplifying assumption every orbit with $\chi_1 > 2$ will be considered trapped and every one with $\chi_1 < 2$ will be considered free. If Equation 8 is just barely satisfied (for example $R_1 = 0.9$) it is easily seen from Equations 7 that no trapped orbits penetrate very far into region II. As R_1 decreases this situation changes rapidly until at $R_1 = 1/2$, for *any* r, orbits with $\omega < \pi/2$ are trapped and those with $\omega > \pi/2$ are free. This is obviously when the anisotropy is most pronounced.

By assuming that the daily variation indeed arises in this fashion and that it is most pronounced at $p = 15$ Bev/c, Equation 6 gives

$$R_1 = 350\, R_0 \quad .$$

If the solar radius is chosen for R_0, then R_1 is approximately 2 astronomical units — considerably less than is generally believed to be its value, but not an impossible value (for discussion and references see the analysis by Axford, Dessler, and Gottlieb in Reference 10).

This explanation has two fundamental difficulties. First, the polar field of the sun was observed to reverse its direction during the solar maximum of 1958 (Reference 11) while the cosmic ray anisotropy maintained its direction. It has been suggested that the sun's polar field is not the main source of the interplanetary magnetic field and that the latter does not reverse (Reference 5). In any case, it is hoped that this point will be resolved by future observations. The second difficulty is that according to this explanation, the anisotropy occurs only in a very narrow energy band; it does not explain, for instance, the observation of the daily variation underground (Reference 12). It is possible, however, that a more realistic (and less abrupt) model of the outer boundary will resolve this problem.

THE DENSITY GRADIENT MODEL

The density gradient model was described by Dattner and Venkatesan (Reference 13) and worked out in detail by Elliot (References 5 and 6). One of the basic assumptions is that the interplanetary magnetic field in the vicinity of the earth is perpendicular to the ecliptic; of course, this does not agree with the radial stretching of magnetic lines of force by the solar wind, but this point will not be considered now. In this discussion r will be the distance from the sun to an observer on earth. Consideration will be given only to particles with momentum p, which will have a gyration radius $a(r)$ in the earth's vicinity. Particles arriving at the earth's orbit tangentially from one direction will then have their guiding center at the distance $r + a$, and those arriving from the opposite direction will have it at $r - a$. If a sunward gradient exists in the flux density Φ (reckoned at the guiding center of the particles it describes), the fluxes in the two directions are not equal and their ratio to the first order in a/r is $(1 + \delta)$, where (Reference 5)

$$\delta = \frac{2a}{\Phi}\frac{d\Phi}{dr} . \tag{9}$$

There is good reason to believe a density gradient actually exists in interplanetary space, since the flux density arriving at the earth undergoes a modulation connected with the solar cycle, and this modulation presumably extends only a finite distance from the sun. A different question is whether the gradient is pronounced in the vicinity of the earth's orbit. No evidence of an appreciable gradient was found (References 14 and 15) by either Pioneer V (1960 $\alpha 1$) or Mariner II (1962 $\alpha\rho 2$); however, the radiation detectors aboard both these space probes were sensitive down to energies below 100 Mev, so that the absence of a density gradient in the energy range in which an anisotropy is observed on the earth may not be considered proven.

A gradient of flux density is not, however, sufficient to create an anisotropy. As a simple illustration, suppose the radiation is acted upon by an electric field due to a positively charged magnetic dipole. In such a field a flux density gradient will exist; but, from Liouville's theorem, if the radiation is isotropic far from the dipole it will be so anywhere in the field (effects of trapping are not considered here). It is instructive to examine the mechanism by which this happens.

We shall consider monoenergetic particles with charge q moving in the symmetry plane of a magnetic dipole field set up around a positive charge at the origin, and assume for simplicity that the motion is nonrelativistic. By Liouville's theorem, with phase-space density τ,

$$\Phi = \frac{\tau}{m} p^3 , \tag{10}$$

$$\frac{1}{\Phi}\frac{d\Phi}{dr} = \frac{3}{p}\frac{dp}{dr} . \tag{11}$$

Equations 10 and 11 refer to the flux density at the observation point; referring instead to the guiding center causes a small correction to be added. In Equation 10 the correction is of the order $\delta/2$ and of course cannot be disregarded, since the entire theory rests on it. In Equation 11, however, the correction is of the second order and will be neglected. Let W be the mean kinetic energy at distance r and $E(r)$ the (radial) electric field intensity there. Then

$$\frac{dp}{dr} = \frac{m}{p}\frac{dw}{dr} = -\frac{m}{p} qE .$$

By substituting $a = p/qB$, we obtain

$$\delta = -\frac{6E}{vB} . \tag{12}$$

On the other hand the electric field also causes the guiding center to drift in the direction of the anisotropy with velocity v_D, which by the nonrelativistic guiding center theory is

$$v_D = \frac{|\mathbf{E} \times \mathbf{B}|}{B^2} = \frac{\mathbf{E}}{B} . \tag{13}$$

In the reference frame of its guiding center, a particle spends equal time moving in any direction in its plane of gyration. Given a large number of particles arriving from infinity, an observer moving with this frame sees an isotropic flux. The flux distribution in a frame of reference moving with velocity v_D relative to a frame of reference in which particles arrive isotropically, has been calculated (for the extreme relativistic limit) by Compton and Getting (Reference 16). For nonrelativistic motion in which v_D is much smaller than the particle velocity v, the flux in the forward direction increases by a factor $1 + 3(v_D/v)$, whereas in the backward direction it decreases by an equal amount. Therefore an anisotropy ratio of $1 + (6E/vB)$ will arise, completely canceling the gradient effect.

More generally, if a density gradient is responsible for the anisotropy, it cannot be caused by a simple potential field, for example by Ψ_0 in the model used here. This is expected to hold even for relativistic particles, for Liouville's theorem remains true at relativistic velocities. It is of course possible that a nonconservative field may exist in the solar system by which particles gain (Reference 17) or lose (Reference 18) energy. Were it not for the radial stretching of the lines of force, such a field could, in principle, explain the anisotropy. In any case the solar cycle modulation and any flux density gradient which might be observed in space may very well be due to a conservative mechanism and have no connection with anisotropies.

ANISOTROPY DUE TO THE COMPTON-GETTING EFFECT

Ahluwalia and Dessler (Reference 19) developed a theory ascribing the anisotropy to relative motion between the earth and a frame of reference in which the cosmic radiation is isotropic. The orbital motion of the earth, for instance, would produce such an effect. However this would (Reference 13) have a phase opposite to what is observed and an amplitude of only 0.03 percent. In this theory the sun is assumed to be surrounded by matter flowing radially outwards, as in the model used here. An electric field is then set up which causes cosmic ray particles to drift across it and be isotropic in a frame of reference moving with the drift velocity. An earlier theory of this kind, by Brunberg and Dattner (Reference 20), assumes the electric field is created by co-rotation of the interplanetary gas with the sun, extending at least to the earth's orbit.

In a highly conducting ionized gas an electric field will indeed exist, tending to the limiting value of $-(\mathbf{v} \times \mathbf{B})$. However, if $\mathbf{B}$ is axisymmetric around the rotation axis,

$$\mathrm{curl}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = 0 ,$$

so that E is conservative, and according to the conclusions of the previous section no anisotropy arises.

The possibility remains that the field is not symmetric around the solar rotation axis [for example, because of "beams" of enhanced velocity as suggested by Alfvén (Reference 21)]. For that case, however, it is hard to explain the constancy of the direction of the anisotropy. If the field is increasing at a certain time, creating an anisotropy in the observed direction, several days later the field will be dropping to its previous value, so that particles in those orbits in which acceleration took place in the first instance will now be decelerated. Then the anisotropy would be expected to reverse direction, or at least undergo a considerable change in phase. Also, a much better correlation would be expected than that observed between the amplitude of the anisotropy and solar disturbances.

CONCLUSION

It can be seen from the preceding that all three explanations of the cosmic ray anisotropy meet serious difficulties. Although no detailed description should be expected from the crude model used, the following general conclusions may be drawn:

1. Anisotropies due to trapped orbits are possible in the stretched dipole field, but only in a narrow energy range depending on the strength of the field's source and on the distance at which the lines of force begin closing. Of course, no anisotropy will be observed unless a preferential loss mechanism for trapped orbits exists.

2. The existence, or lack of existence, of a radial flux density grandient in interplanetary space may be totally unrelated to the observed anisotropy.

3. A conservative electric field, as proposed by Dessler and Ahluwalia, will not give rise to an anisotropy.

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